"I don't like it, and I'm sorry I ever had anything to do with it."

28th April, 1926
Schrödinger to Einstein in a letter on "wave mechanics"

Why did Schrödinger dislike quantum mechanics?

- It makes statistical predictions, determinism is lost. PROBABILITY
- Did not like wave-function collapse, and the measurement problem.
- Did not like the implications for "reality". MEASUREMENT.

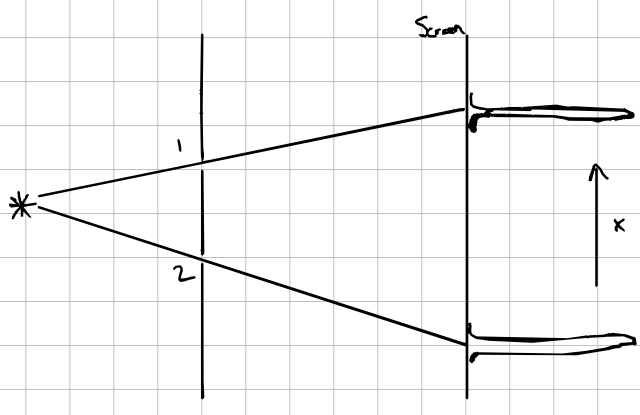
Is light made of waves or particles?

Newton (1704) argued for particles, Huygens (1678) for waves.

Young's (1801) double slit experiment resolved the argument in favour of waves.

Waves vs. Particles

Particle beams:



Two peaks.

Particles are fired one by one and are detected on the screen $n_{12}(x)$ are counted at x. The peaks are spread a little because nothing is perfect. If N are fired we can find a probability of detection at x

$$P_{12}(x) = n_{12}(x)/N.$$

If we close hole 2, we can count $n_1(x)$ and find

$$P_1(x) = n_1(x)/N.$$

If we close hole 1, similarly

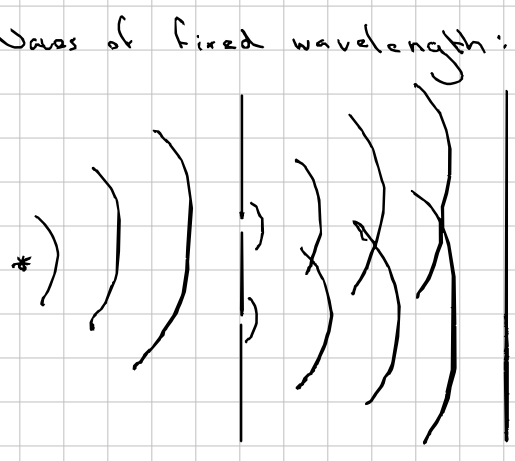
$$P_2(x) = n_2(x)/N.$$

Since a particle goes through either hole 1 or hole 2 then $n_{12}(x) = n_1(x) + n_2(x)$ and

$$P_{12}(x) = P_1(x) + P_2(x).$$

Probabilities add for particles.

Waves of fixed wavelength:



What do we see?

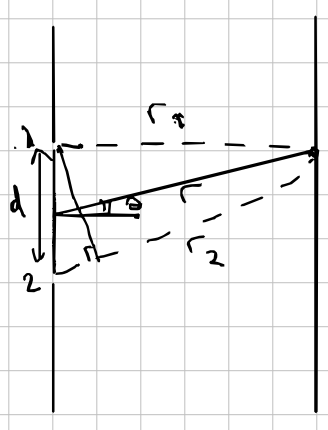
Bright spots if wave peaks arrive together.
Dark spots if a wave peak coincides with a wave trough.

Real, harmonic waves $\cos(kx - \omega t)$, $\sin(kx - \omega t)$

$k = 2\pi/\lambda$ $\omega = 2\pi/T = 2\pi f$.

can be written via Euler's formula as a complex wave:

$$Ae^{i(kx-\omega t)} = A(\cos(kx-\omega t) + i\sin(kx-\omega t))$$



Roughly (for small θ),

$$r_1 - r_2 \simeq d\sin\theta \equiv \delta$$

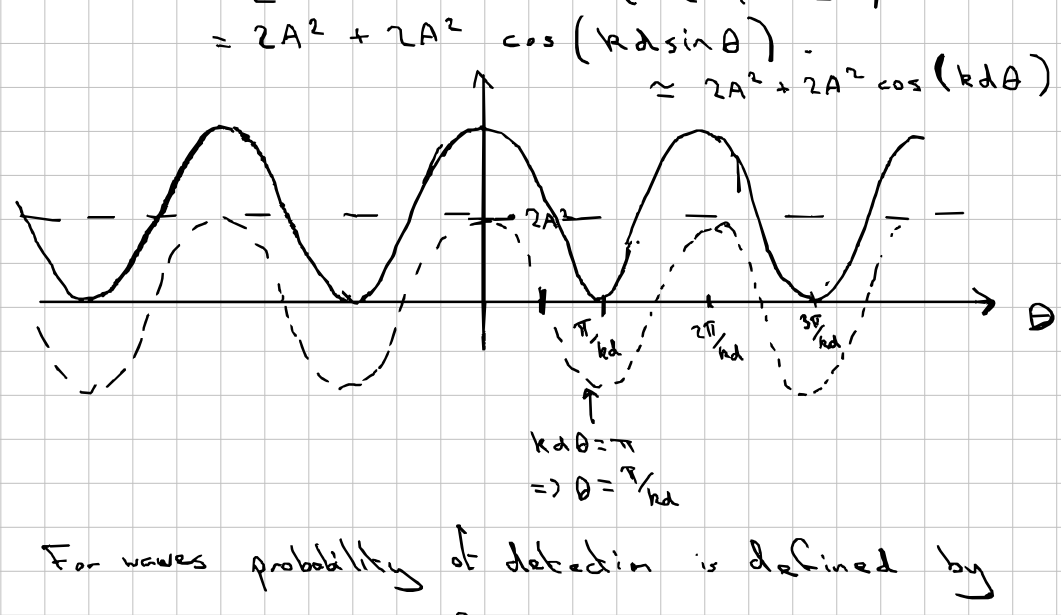
Wave amplitudes add, and we measure waves from their amplitudes displacing things. $I \propto A^2$

On the screen

$$\psi_1 + \psi_2 = Ae^{i(kr_1-\omega t)} + Ae^{i(kr_2-\omega t)}$$

So

$$\begin{aligned} I &= |\psi_1 + \psi_2|^2 \\ &= (\psi_1 + \psi_2)(\psi_1 + \psi_2)^* \\ &= |\psi_1|^2 + \psi_1\psi_2^* + \psi_2\psi_1^* + |\psi_2|^2 \\ &= 2A^2 + 2A^2 Re(\psi_1\psi_2^*) \\ &= 2A^2 + 2A^2\cos(k(r_1 - r_2)) \\ &= 2A^2 + 2A^2\cos(kd\sin\theta) \\ &\simeq 2A^2 + 2A^2\cos(kd\theta) \end{aligned}$$



For waves probability of detection is defined by

$$P(x) = |\psi(x)|^2$$

Hole 2 closed,

$$P_1(x) = |\psi_1(x)|^2$$

Hole 1 closed,

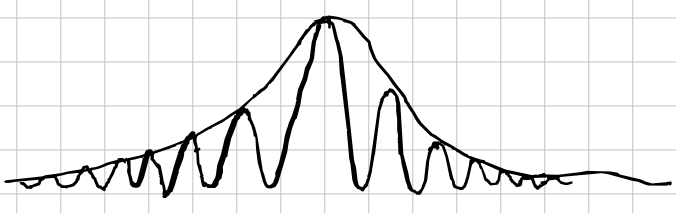
$$P_2(x) = |\psi_2(x)|^2$$

Then

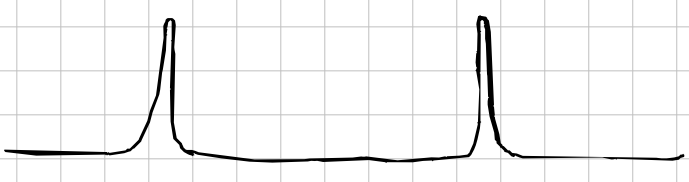
$$\begin{aligned} P_{12}(x) &= |\psi_1(x) + \psi_2(x)|^2 \\ &= |\psi_1|^2 + |\psi_2|^2 + 2Re(\psi_1\psi_2^*) \\ &= P_1(x) + P_2(x) + \text{Interference terms.} \end{aligned}$$

Probabilities for waves have interference terms.

What we find is



Unless we turn on a detector and then we find



Light is behaving like something new: it is both wave and particle, it is called the quantum particle.

The Old Quantum Theory.

1900 ~ 1924 More evidence of particle like behaviour
Planck
for light and energy:

$E = nh\nu$ (ν: Frequency)

Planck's constant

$h \simeq 6.626\ldots \times 10^{-34}\ J\,s$, $\frac{h}{2\pi} = \hbar$.

1905 Einstein photoelectric effect
1923 Compton scattering
} Photons of light.

1911 Rutherford's model of the atom — unstable
Accelerating electrons radiate energy.

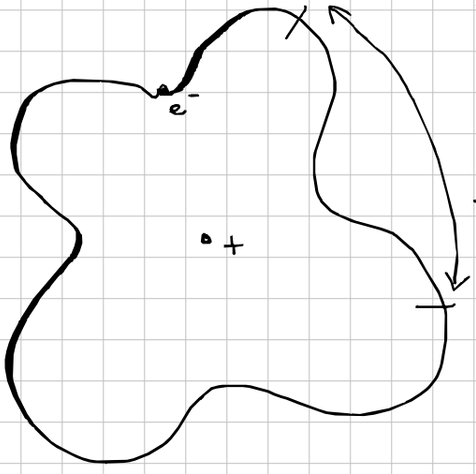
1913 Bohr stable because energy is quantised and only released in lumps; so electrons can't spiral into the nucleus. $L = n\frac{h}{2\pi} \equiv n\hbar$ $= mvr$

1923 - de Broglie if light waves can be particles can particles be understood as waves?

$E = h\nu$

$p = h/\lambda$

$E = pc = h\nu$
$p = h\nu/c$ $\lambda = c/\nu$
$= h/\lambda$.



$2\pi r = n\lambda = nh/p$

$= nh/mv$

$= nhr/L$

$L = mvr$

$\frac{1}{mv} = r/L$

$\rightsquigarrow$ $2\pi = nh/L$

or $L = nh/2\pi = n\hbar$.

$\rightsquigarrow$ electrons behave like waves!

(1927) Davisson and Germer showed that electrons produced interference patterns.

The Schrödinger Equation.

de Broglie $E = h\nu$, $p = h/\lambda$

Waves: $Ae^{i(kx - \omega t)}$

$k = 2\pi/\lambda$, $\omega = \frac{2\pi}{T} = 2\pi\nu$

$\therefore \quad k = 2\pi\left(1/\lambda\right) = 2\pi\left(p/h\right) = p/\hbar$

$\omega = 2\pi\nu = 2\pi\left(E/h\right) = E/\hbar$.

Matter waves:

$$\psi \equiv A\, e^{i(px - Et)/\hbar} \qquad -(*)$$

For a free particle classical physics says

$$E = \tfrac{1}{2}mv^2 = p^2/2m \qquad p = mv.$$

How do we get this equation from $(*)$?

We need to know that $\frac{d}{dx}\left(e^{ax}\right) \equiv a e^{ax}$

then $\frac{d\psi}{dt} = -\frac{iE}{\hbar} A e^{i(px - Et)/\hbar}$

$= -\frac{iE}{\hbar}\psi$

and $\frac{d\psi}{dx} = A\frac{ip}{\hbar} e^{i(px - Et)/\hbar}$

$= \frac{ip}{\hbar}\psi. \quad \Rightarrow \frac{d^2\psi}{dx^2} = -\frac{p^2}{\hbar^2}\psi$

$\therefore \quad E\psi = i\hbar\frac{d\psi}{dt}$ and $p^2 = -\hbar^2\frac{d^2\psi}{dx^2}$

i.e. $E\psi = \frac{p^2}{2m}\psi \iff i\hbar\frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}$.

$$i\hbar\frac{d\psi}{dt} = \left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\psi$$

$$\equiv \hat{H}\psi.$$

But what is ψ?

Born: it is a probability distribution.

Comment: The Schrödinger equation is a linear, homogeneous equation: superposition.

ψ contains information about all possible outcomes.

$$\psi = \sum_{i=1}^{n} a_i \varphi_i$$

We find outcome φ_i with probability $|a_i|^2$.

Heisenberg's uncertainty relation:

$$\Delta x \, \Delta p \geq \hbar/2.$$

Measurement

The wavefunction

$$\Psi(x,t) = \sum_i a_i \Psi_i$$

contains a probability $|a_i|^2$ for each possible outcome Ψ_i

Before a measurement all outcomes are possible, afterwards the probabilities are changed to reflect the measurement

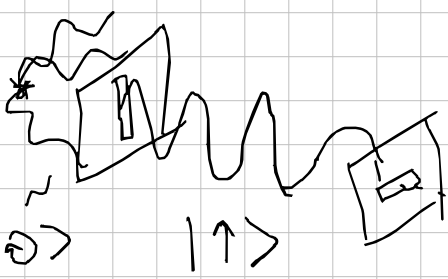
If we measure that a system is in state Ψ_i then the wavefunction collapses:

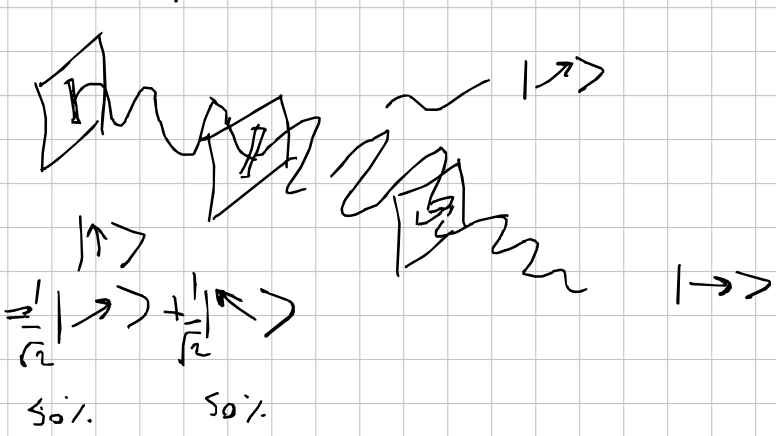
$$\Psi \longrightarrow \Psi_i$$

In the double slit experiment

$\Psi_1 + \Psi_2 \rightarrow \Psi_1$ if we know the photon / electron goes through hole 1.

Polarized light

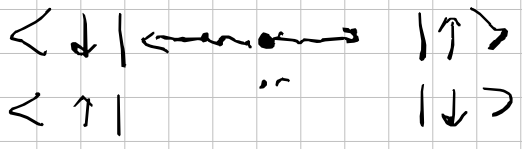




Schrödinger's cat.

What is a measurement?

Teleportation & spooky action at a distance.



$$\Psi = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B \right)$$

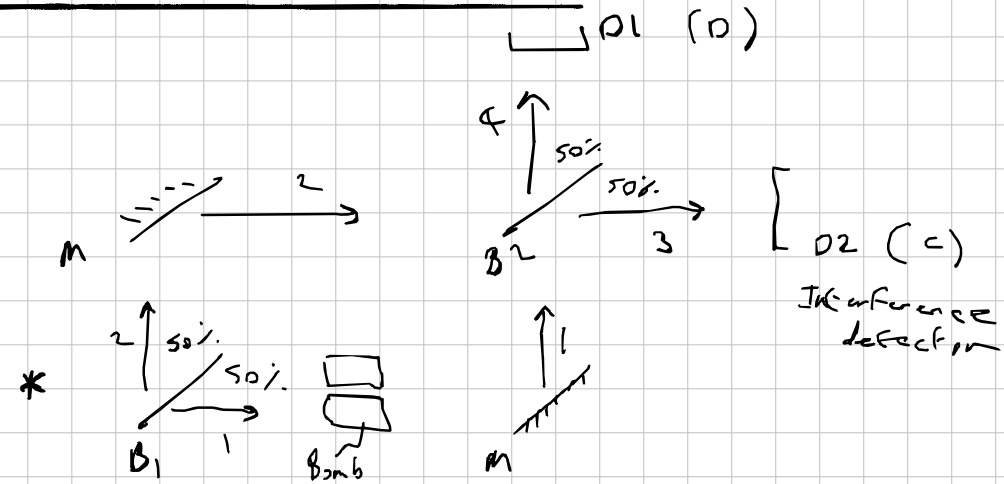
A measures Ψ and knows what result B will get.

How to use this?

Send a codebook, physically.

No cloning.

Elitzur - Vaidman Bomb Tester.



Setup: no bomb ⇒ detection at D2
(dud bomb) constructive interference

no detection at D1

Live bomb:

½ Path 1: → explosion.

½ Path 2: → no interference → ½ D1

½ D2.

Anything at D1 implies alive bomb, but not exploded.